(01)

01 - a,b,c,d   X   Y

$2X + Y$ $X + 3Y$ $2Y-3X$

$$Y = \begin{pmatrix} 3 & -5 \\ c & 3d-2 \end{pmatrix} \qquad X = \begin{pmatrix} a-2 & 2b+1 \\ a+3 & 16 \end{pmatrix}$$

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$a-2=3 \Rightarrow a=5 \qquad 2b+1=-5 \Rightarrow b=-3 \qquad 3d-2=16 \Rightarrow d=6 \qquad a+3=c \Rightarrow c=8$

وبالتالي تكون

$$Y = \begin{pmatrix} 3 & -5 \\ 8 & 16 \end{pmatrix} \qquad X = \begin{pmatrix} 3 & -5 \\ 8 & 16 \end{pmatrix}$$

$$2X + Y = \begin{pmatrix} 6 & -10 \\ 16 & 32 \end{pmatrix} + \begin{pmatrix} 3 & -5 \\ 8 & 16 \end{pmatrix}$$

$$2X + Y = \begin{pmatrix} 3+6 & -10-5 \\ 16+8 & 32+16 \end{pmatrix} = \begin{pmatrix} 9 & -15 \\ 24 & 48 \end{pmatrix}$$

$$X + 3Y = \begin{pmatrix} 3 & -5 \\ 8 & 16 \end{pmatrix} + \begin{pmatrix} 9 & -15 \\ 24 & 48 \end{pmatrix}$$

$$X + 3Y = \begin{pmatrix} 3+9 & -5-15 \\ 8+24 & 16+48 \end{pmatrix} = \begin{pmatrix} 12 & -20 \\ 32 & 64 \end{pmatrix}$$

$$3X - 2Y = \begin{pmatrix} 9 & -15 \\ 24 & 48 \end{pmatrix} - \begin{pmatrix} -6 & 10 \\ -16 & -32 \end{pmatrix}$$

$$3X - 2Y = \begin{pmatrix} 9-6 & -15+10 \\ 24-16 & 48-32 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 8 & 16 \end{pmatrix}$$

$$\begin{pmatrix} 4X_1 + 5X_2 + 3X_3 = 77 \\ 5X_1 + X_2 + 2X_3 = 63 \\ 3X_1 + 2X_2 + X_3 = 44 \end{pmatrix}$$

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$$\begin{cases} 5X_1 + X_2 + 2X_3 = 63 \\ 4X_1 + 5X_2 + 3X_3 = 77 \\ 3X_1 + 2X_2 + X_3 = 44 \end{cases} \Leftrightarrow \begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ 4X_1 + 5(63 - 5X_1 - 2X_3) + 3X_3 = 77 \\ 3X_1 + 2(63 - 5X_1 - 2X_3) + X_3 = 44 \end{cases}$$

$$\begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ 4X_1 + 315 - 25X_1 - 10X_3 + 3X_3 = 77 \\ 3X_1 + 126 - 10X_1 - 4X_3 + X_3 = 44 \end{cases} \Leftrightarrow \begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ -21X_1 - 7X_3 = -238 \\ -7X_1 - 3X_3 = -82 \end{cases}$$

$$\begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ X_3 = 34 - 3X_1 \\ -7X_1 - 3(34 - 3X_1) = -82 \end{cases} \Leftrightarrow \begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ X_3 = 34 - 3X_1 \\ -7X_1 - 102 + 9X_1 = -82 \end{cases}$$

$$\begin{cases} X_2 = 63 - 5X_1 - 2X_3 \\ X_3 = 34 - 3X_1 \\ 2X_1 = 20 \end{cases} \Leftrightarrow \begin{cases} X_2 = 63 - 5(10) - 2X_3 \\ X_3 = 34 - 3(10) \\ X_1 = 10 \end{cases} \Leftrightarrow \begin{cases} X_2 = 5 \\ X_3 = 4 \\ X_1 = 10 \end{cases}$$

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$$\begin{pmatrix} 4X_1 + 5X_2 + 3X_3 = 77 \\ 5X_1 + X_2 + 2X_3 = 63 \\ 3X_1 + 2X_2 + X_3 = 44 \end{pmatrix} = \overset{A}{\begin{pmatrix} 4 & 5 & 3 \\ 5 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}} \; X \; \overset{X}{\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}} = \overset{b}{\begin{pmatrix} 77 \\ 63 \\ 44 \end{pmatrix}}$$

$$Det\,A = \begin{vmatrix} 4 & 5 & 3 \\ 5 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 4(-3) - 5(-1) + 3(7) = 14$$

$$A' = \begin{pmatrix} -3 & 1 & 7 \\ 1 & -5 & 7 \\ 7 & 7 & -21 \end{pmatrix}$$

$$A^{-1} = (1/14) \begin{pmatrix} -3 & 1 & 7 \\ 1 & -5 & 7 \\ 7 & 7 & -21 \end{pmatrix}$$

$$X = A^{-1} . b = \begin{pmatrix} -3 & 1 & 7 \\ 1 & -5 & 7 \\ 7 & 7 & -21 \end{pmatrix} X \begin{pmatrix} 77 \\ 63 \\ 44 \end{pmatrix} = \begin{pmatrix} (-231/14) + (63/14) + 22 \\ (77/14) - (315/14) + 22 \\ (77/2) + (63/2) - (132/2) \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \\ 4 \end{pmatrix}$$

$$Det\ X_1\ =\ \begin{vmatrix} 77 & 5 & 3 \\ 63 & 1 & 2 \\ 44 & 2 & 1 \end{vmatrix}\ =\ 77(-3)\ -5(-25)\ +3(82) = 140$$

$$X_1\ =\ (140/14) \Leftrightarrow\ X_1\ =\ 10$$

$$Det\ X_1\ =\ \begin{vmatrix} 4 & 77 & 3 \\ 5 & 63 & 2 \\ 3 & 44 & 1 \end{vmatrix}\ =\ 4(-25)\ -77(-1)\ +3(312) = 70$$

$$X_2\ =\ (70/14) \Leftrightarrow\ X_2 = 5$$

$$Det\ X_1\ =\ \begin{vmatrix} 4 & 5 & 77 \\ 5 & 1 & 63 \\ 3 & 2 & 44 \end{vmatrix}\ =\ 4(-82)\ -5(-31)\ +7(77) = 56$$

$$X_3\ =\ (56/14) \Leftrightarrow\ X_3\ =\ 4$$