$$:$$

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$$

$$
\begin{array}{cccccc}
 & n & & m & & \\
a_{11} x_1 & |\, a_{12} x_2 & \cdots & |\, a_{1n} x_n & = & b_1 \\
a_{21} x_1 & +a_{22} x_2 & \cdots & +a_{2n} x_n & = & b_2 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \\
a_{m1} x_1 & +a_{m2} x_2 & \cdots & +a_{mn} x_n & = & b_m
\end{array}
$$

$$:$$

$$Ax = b$$

$$
A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}
$$

المصفوفات

$$:$$

$$
A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}
$$

a (a_{ij}) n   m   n   m

(j)   (i)

مثال:

$$
\begin{pmatrix} 4X_1 + 5X_2 + 3X_3 = 77 \\ 5X_1 + X_2 + 2X_3 = 63 \\ 3X_1 + 2X_2 + X_3 = 44 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 3 \\ 5 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix} \; X \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 77 \\ 63 \\ 44 \end{pmatrix}
$$

$$A \qquad X \quad = \quad b$$

-

-

Cramer   -

$$\begin{cases} 2x_1 + 3x_2 = 9 \\ x_1 - x_2 = 2. \end{cases}$$

:

:   -

$2X_1 + 3X_2 = 9$ .... (1)
$X_1 - X_2 = 2$ .... (2)

(X₂)   (X₁)   (2)

$X_1 = 2 + X_2$ .... (3)

:   (1)   (3)   (X₁)

$2(2 + X_2) + 3X_2 = 9 \Rightarrow 4 + 2X_2 + 3X_2 = 9$
$\Rightarrow 5X_2 = 5 \Rightarrow \mathbf{X_2 = 1}$........................... (4)

:   (3)   (4)   (X₂)

$X_1 = 2 + X_2 \Rightarrow X_1 = 2 + 1 \Rightarrow \mathbf{X_1 = 3}$

:   -

: $Ax = b$

$$b = \begin{pmatrix} 9 \\ 2 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad A = \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$$

A^{-1}

:

$$\det A = -2 - 3 = -5$$

:

$$A^{-1} = \frac{1}{\det A} \begin{pmatrix} -1 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{5} & \dfrac{3}{5} \\ \dfrac{1}{5} & -\dfrac{2}{5} \end{pmatrix}$$

:

$$x = A^{-1}b = \begin{pmatrix} \dfrac{1}{5} & \dfrac{3}{5} \\ \dfrac{1}{5} & -\dfrac{2}{5} \end{pmatrix} \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$(X_2 = 1)$ $(X_1 = 3)$:

.

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad \Delta x_1 = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad \cdots$$

.

$$x_1 = \frac{\Delta x_1}{\Delta}, \quad x_2 = \frac{\Delta x_2}{\Delta}, \quad \cdots$$

:   :

$$\begin{cases} 2x_1 + 3x_2 = 9 \\ x_1 - x_2 = 2 \end{cases}$$

-

$$\Delta \quad \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = \quad 2(-1) - 1(3) \quad -5$$

-

$$\Delta X_1 \quad \begin{vmatrix} 9 & 3 \\ 2 & -1 \end{vmatrix} = \quad 9(-1) - 2(3) \quad -15$$

-

$$\Delta X_1 \quad \begin{vmatrix} 2 & 9 \\ 1 & 2 \end{vmatrix} = \quad 2(2) - 1(9) \quad -5$$

:   -

$$X_1 = \frac{\Delta X_1}{\Delta} = \frac{-15}{-5} = 3$$

$$X_2 = \frac{\Delta X_2}{\Delta} = \frac{-5}{-5} = 1$$

: $(X_1 = 3)$ $(X_2 = 1)$

:   :

تمرين

:

$$\begin{pmatrix} X = 3 - Y \\ 6X + Y + 2 = 0 \end{pmatrix} \qquad \begin{pmatrix} Y + 4X = 2 \\ X - 2Y = 1 \end{pmatrix} \qquad \begin{pmatrix} Y = X + 2 \\ 4X + 3 = Y \end{pmatrix}$$